Physics Letters B 748 (2015) 9–12

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Finite size of hadrons and Bose–Einstein correlations in *pp* collisions at 7 TeV

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ARTICLE INFO

Article history:
Received 16 March 2015
Received in revised form 10 June 2015
Accepted 22 June 2015
Available online 26 June 2015
Editor: J.-P. Blaizot

Keywords:
Proton–proton collisions
Bose–Einstein correlations

ABSTRACT

Space–time correlations between produced particles, induced by the composite nature of hadrons, imply specific changes in the properties of the correlation functions for identical particles. The expected magnitude of these effects is evaluated using the recently published blast-wave model analysis of the data for *pp* collisions at $\sqrt{s} = 7$ TeV.

1. It has been recently pointed out [1] that since hadrons produced in high-energy collisions are not point-like objects, they cannot be uncorrelated. Indeed, being composite, hadrons cannot occupy too close space–time points (because at small distance the constituents of hadrons mix and there are no separate hadrons to interfere). Consequently, since the HBT experiment measures the quantum interference between wave functions of hadrons, it cannot see hadrons which are too close to each other. Therefore the distribution function of the pair of hadrons must vanish at small distances between them.

This implies of course a correlation in space–time. As this correlation is the *necessary* consequence of the composite structure of hadrons (and thus it is a general property of the system) it is interesting to investigate to what extent it modifies the accepted ideas about the quantum interference which are, usually, derived under the assumption that such correlations can be neglected [2].

It was already shown in [1] that such space–time correlations may be responsible for the observation that the two-pion Bose–Einstein correlation function takes values below unity [3–5], at variance with the well-known theorem valid when the correlations are ignored [2].

In the present paper the investigation of this phenomenon is continued, using the recently published [6] analysis of the data on HBT radii, measured by the ALICE Collaboration [7]. This allows to estimate quantitatively the magnitude of the effect and to give predictions for its size in all three directions, *long*, *side* and *out*, commonly used in discussion of the quantum interference [2].

In the next section the consequences of the space–time correlations for the HBT correlation functions are explained. In Sections 3 and 4 the blast-wave model used for the quantitative estimate of the effect is presented. The results are presented and summarized in the last two sections.

2. In absence of correlations between produced hadrons, the two-particle source function is the simple product

$$w(p_1, p_2; x_1 x_2) = w(p_1, x_1) w(p_2, x_2) \tag{1}$$

where $w(p, x)$ is the single-particle source function (Wigner function). Consequently, the Bose–Einstein correlation function between the momenta of two identical particles

$$C(p_1, p_2) \equiv \frac{N(p_1, p_2)}{N(p_1) N(p_2)} \tag{2}$$

is given by [2]

$$\begin{aligned} C(p_1, p_2) &= 1 + \frac{\tilde{w}(P_{12}; Q)\tilde{w}(P_{12}; -Q)}{w(p_1)w(p_2)} \\ &= 1 + \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} \geq 1. \end{aligned} \tag{3}$$

Here

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http://dx.doi.org/10.1016/j.physletb.2015.06.053
0370-2693/

$$\tilde{w}(P_{12};Q)=\int dx\, e^{iQx}w(P_{12};x),$$
$$w(p)=\int dx\, w(p;x), \tag{4}$$

where $P_{12}=(p_1+p_2)/2$ and $Q=p_1-p_2$.

The data from the L3 Collaboration [3] and from the CMS Collaboration [5] show that the correlation function $C(p_1,p_2)$ takes values below unity, contrary to Eq. (3). Thus the particles must be correlated and we propose that this effect is due to the composite nature of hadrons.

To implement these space–time correlations, we replace formula (1) for the two-particle source function by

$$W(p_1,p_2;x_1,x_2)=w(p_1;x_1)w(p_2;x_2)[1-D(x_1-x_2)], \tag{5}$$

where $D(x_1-x_2)$ is the cut-off function that satisfies the constraint $D(x_1-x_2=0)=1$ and tends to 0 at larger distances (above, let us say, 1 fm). Then, the HBT correlation function becomes

$$C(P_{12},Q)=C_{\rm noncorr}(P_{12},Q)-C_{\rm corr}(p_1,p_2), \tag{6}$$

where the uncorrelated part $C_{\rm noncorr}(P_{12},Q)$ is given by (3), while the correction due to space–time correlations reads

$$C_{\rm corr}=C^{(0)}_{\rm corr}+C^{(Q)}_{\rm corr} \tag{7}$$

where

$$C^{(0)}_{\rm corr}=\frac{\int dx_1dx_2\, w(p_1;x_1)w(p_2;x_2)D(x_1-x_2)}{w(p_1)w(p_2)}, \tag{8}$$

$$C^{(Q)}_{\rm corr}=\frac{\int dx_1dx_2 e^{i(x_1-x_2)Q}\, w(P_{12};x_1)w(P_{12};x_2)D(x_1-x_2)}{w(p_1)w(p_2)}. \tag{9}$$

One sees that the contribution from the correlation part is negative. Moreover, since it obtains contributions from a small region of space–time, its dependence on Q is much less steep than that of the uncorrelated part. Consequently, at Q large enough $C(P_{12},Q)$ may easily fall below one.

To describe the actual measurements one has to take into account that particles produced very far from the center (e.g. those arising from long-lived resonances) form a "halo" and do not contribute to the HBT correlations [8]. Thus we have

$$\hat{C}_{\rm obs}(P_{12},Q)=1-p^2+p^2C(P_{12},Q) \tag{10}$$

where p^2 is the probability that both particles originate from the "core".

In the ALICE experiment [7] $\hat{C}_{\rm obs}$ was, in addition, normalized to 1 at some Q_0 where the influence of quantum interference is expected to be negligible. Thus we finally have to consider the function

$$C_{\rm obs}(P_{12},Q)=\frac{1-p^2+p^2C(P_{12},Q)}{1-p^2+p^2C(P_{12},Q_0)}. \tag{11}$$

Introducing the (measured) intercept parameter λ by the condition

$$1+\lambda\equiv C_{\rm obs}(P_{12},Q=0) \tag{12}$$

one obtains

$$p^2=\frac{\lambda}{C(P_{12},Q=0)-C(P_{12},Q_0)+\lambda[1-C(P_{12},Q_0)]}. \tag{13}$$

This allows to evaluate the measured correlation function in terms of the measured intercept parameter λ and the evaluated correlation function $C(P_{12},Q)$.

Note that in absence of space–time correlations we have $C(P_{12},Q=0)=2$, $C(P_{12},Q_0)=1$, and thus $p^2=\lambda$, as is usually assumed.

3. To have an idea on the magnitude of the effect we discuss, we have used the blast-wave model described in detail in [6,9]. In this model, at freeze-out, hadrons are created at a fixed (longitudinal) proper time

$$\tau\equiv\sqrt{t^2-z^2}=\tau_f. \tag{14}$$

The single-particle source function (in the longitudinal c.m.s. system) becomes

$$w(p,x)=k_0\cosh\eta e^{-U\cosh\eta+V\cos\phi}f(r)rdrd\phi d\eta \tag{15}$$

where $k_0=\sqrt{m^2+k_\perp^2}$, whereas η, ϕ and r are space–time rapidity, azimuthal angle and transverse distance from the symmetry axis.[1] We have also introduced the notation

$$U=\beta k_0\cosh\theta;\quad V=\beta k_\perp\sinh\theta, \tag{16}$$

with $T=1/\beta$ being the freeze-out temperature. Finally, θ describes the transverse flow by the relation

$$\sinh\theta=\omega r, \tag{17}$$

with ω being a parameter. The function $f(r)$ describes the transverse profile of the source.

It was shown in [6] that the model is flexible enough to describe the HBT radii measured by the ALICE Collaboration [7]. The function $f(r)$ was taken in the form

$$f(r)\sim e^{-(r-R)^2/\delta^2} \tag{18}$$

corresponding to a "shell" of the width $\sqrt{2}\delta$ and radius R.

Thus the model contains 5 free parameters: T, ω, τ_f, R and δ, which may depend on the multiplicity of the event. Their values, giving a good description of the HBT radii measured in [7], are given in [6].

4. Since we treat particles as extended objects produced on the hyperbola (14), the longitudinal distance between the two hadrons located at the space–time rapidities η_1, η_2 should be calculated along this curve, which yields

$$d_\parallel=\int_{\eta_1}^{\eta_2}\sqrt{dz^2-dt^2}=\tau_f(\eta_2-\eta_1). \tag{19}$$

In the frame where $\eta_1+\eta_2=0$ we also have $t_1=t_2$ and thus the total distance between particles is

$$d^2=(x_1-x_2)^2+(y_1-y_2)^2+d_\parallel^2\equiv d_\perp^2+d_\parallel^2. \tag{20}$$

Since this expression is invariant under boost in the longitudinal direction, it is also valid in the LCMS system, and thus we finally have

$$d^2(x_1,x_2)=r_1^2+r_2^2-2r_1r_2\cos(\phi_1-\phi_2)+\tau_f^2(\eta_1-\eta_2)^2. \tag{21}$$

The correlation functions were studied using a Gaussian cut-off function

$$D(x_1,x_2)=e^{-d(x_1,x_2)^2/\Delta^2}, \tag{22}$$

where Δ is a constant fixing the scale of the cut-off region.

[1] All irrelevant constants are cancelled in the definition of $w(p,x)$.

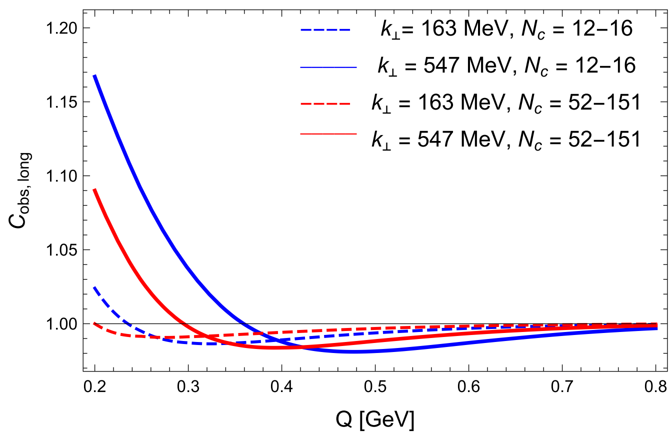

Fig. 1. (Color online.) Correlation function C_{obs} for the *long* direction in the interval 0.2 GeV $\le Q \le 0.8$ GeV (normalized to 1 at $Q = 1$ GeV). The dashed lines describe the results for $k_\perp = 163$ MeV and the two multiplicity classes: $N_c = 12$–16 and $N_c = 52$–151. The solid lines describe the results for $k_\perp = 547$ MeV and the same two multiplicity classes.

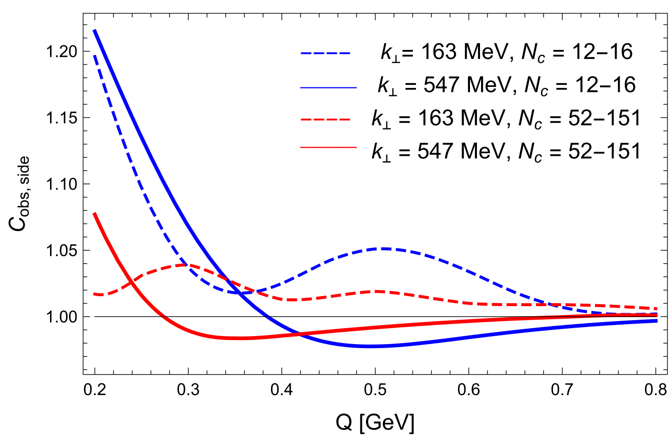

Fig. 2. The same as Fig. 1 but for the side direction.

5. Using (22), (21) and the source function of the model described in Section 3, with the parameters taken from [6],[2] we have evaluated corrections to the HBT correlation functions (6) for all intervals of the particle multiplicity and transverse momentum (as measured in [7]), and for all three directions of the vector $\vec{Q}$. The cut-off distance $\Delta \approx 2r_V$ (where r_V is the radius of the "excluded volume" [10] occupied by one pion) was taken to be 1 fm, within the range of values given by the earlier analyses [11]. Some of the results, obtained using the Gaussian $D(x_1 - x_2)$, are shown in Figs. 1–3.

One sees that for the *long* direction the correlation function falls below 1 at all multiplicities and transverse momenta of the pair. The depth of the minimum in the *long* direction varies from ~0.02 to ~0.01 (below 1) when the HBT radius R_{long} increases from ~0.8 to ~2 fm.

In the *side* and *out* directions the results are strongly dependent on the value of the transverse momentum of the pair. At $k_\perp \le$ 300 MeV for the *side* and $k_\perp \le 400$ MeV for the *out* direction the correlation functions are always larger than 1 in the investigated region. In the *side* direction the correlation function shows a clear structure: a minimum followed by a maximum (particularly at low

[2] The intercept parameter λ was taken as $\lambda = 0.59 - 0.26k_\perp$ (where $k_\perp$ is in GeV) which approximates the data of [7].

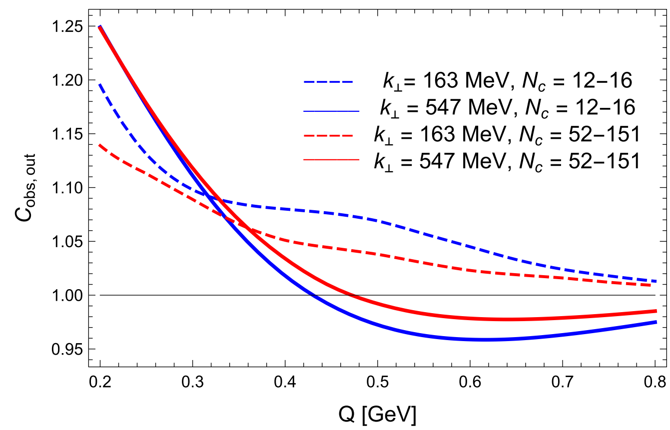

Fig. 3. The same as Fig. 1 but for the out direction.

multiplicities). At larger $k_\perp$ the minimum below 1 shows up in both cases.

In the *side* direction the minimum at $k_\perp \ge 300$ MeV is similar to that found in the *long* direction. It is about twice deeper in the *out* direction (above 400 MeV). In both cases the minimum is deeper when the multiplicity increases. Also in this case the change is controlled by the corresponding HBT radii.

To see the sensitivity of these results to the shape of the cut-off function $D(x_1 - x_2)$ we have also considered a sharp cut-off which is drastically different from the Gaussian. We have found that the qualitative features are unchanged, except that the effects of the cut extend to larger values of Q. This, however, happens in the region where these effects are already small and rather hard to measure. Actually, in most cases the results are almost identical,[3] provided that the cut-off parameter is ~0.75 fm. The only exception is the *side* direction at small multiplicity where the difference exceeds slightly 0.02 at $Q > 700$ MeV.

We thus conclude that although the shape of the cut-off function can influence the details of our results, the general qualitative features remain unchanged.

6. In summary, we have estimated to what extent the space–time correlations implied by the excluded volume effect modify the HBT correlation functions.

Our conclusions can be formulated as follows:

(i) The space–time correlations induced by the finite size of hadrons lead to a rich structure of the HBT correlation functions, depending on (i) the measurement direction, (ii) multiplicity and (iii) the transverse momentum of the pair.

(ii) The difference between the *long* and the two other directions at small $k_\perp$ is particularly striking.

(iii) At large $k_\perp$ the minimum below 1 shows up in every direction. It is about twice deeper for *out* than for the *long* and *side* directions.

Some comments are in order.

(i) We have found that the modification of the HBT correlation functions are only marginally sensitive to the change of shape of the cut-off function $D(x_1 - x_2)$. This means that the effect we discuss is, in practice, described by a single parameter Δ.

(ii) We have been considering the space–time correlations in the source function of two pions, which are a necessary consequence of their composite nature. Naturally, there might be also

[3] In the relevant region $Q > 300$ MeV they differ by less than 0.01, which is consistent with the expected accuracy of our calculations and also with the present experimental accuracy.

other mechanisms contributing to these correlations (e.g. the final state interaction). In this case the parameter Δ should be considered as an effective cut-off distance which summarizes all contributions. Since our calculations show that the measurable effects on the HBT correlation functions depend mostly on Δ (and not on the shape of the function D) it seems hopeless to try to separate the various contributions.

(iii) In our approach the cut-off function is taken independent of particle density. This approximation seems reasonable because, as shown in [6], particle density at freeze-out changes only by 10% in the range of multiplicities we consider. Moreover, the dominant effect of the changing particle density is expected to be a modification of the single-particle source functions of the two pions contributing to interference rather than of their space–time correlation described by $D(x_1 - x_2)$. It follows that the observable effect of the modification of the cut-off function due to change of particle density is expected to be very small, if any.

(iv) It is interesting to speculate about the size of the effects we discuss in case of heavy ion collisions. Taking the source functions in the transverse direction in form of Gaussians (which is a reasonable approximation for heavy ion collisions) one can easily see that the corrections due to finite size of hadrons fall as $(\Delta/R)^2$ where R is the radius of the system. For PbPb collisions this gives factor $\sim 1/30$ compared to the results shown in this paper, implying that the expected effects are negligible. Similar mechanism should be at work in the longitudinal direction. For smaller systems, as those created in p–Pb collisions, the effects are also expected to be smaller than in pp. Precise estimate would, however, require determination of the source functions (see [6]).

Acknowledgements

We thank Adam Kisiel for useful discussions and for help in understanding the ALICE data. Thanks are also due to Viktor Begun for an instructive discussion on the excluded volume effects. This investigation was supported in part by Polish National Science Center grant with decisions No. DEC-2013/09/B/ST2/00497 (A.B. and K.Z.) and No. DEC-2012/06/A/ST2/00390 (W.F).

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